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Measurement of Multi-Period Income Mobility with contingency tables*

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Abstract

We propose a framework for the measurement of income mobility over several time periods, based on the notion that multi-period mobility amounts to measuring the degree of association between the individuals and the time periods in a contingency table. We provide both indices and a pre-ordering condition for multi-period mobility assessments. We illustrate our approach with an empirical application using the EU-SILC rotating panel dataset.

Keywords: Multi-period mobility, Lorenz curve

JEL classification codes: D31 – D63

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1. Introduction

In a survey on income mobility Fields (2008) writes that "Income mobility means different things to different people…One issue is whether the aspect of mobility of interest is intergenerational or intra-generational…Second, agreement must be reached on an indicator of social or economic status and the choice of recipient unit…Third, the mobility questions asked and our knowledge about mobility phenomena may be grouped into two categories, macro and micro …mobility studies….The first distinction to be drawn is between measures of time independence and measures of movement…The various movement indices in the literature may usefully be categorized into five groupings or concepts…*Positional movement… share movement…non-directional income movement…Directional income movement…Mobility as an equalizer of longer-term incomes…"*. This long citation shows indeed how complex the notion of income mobility is.

The focus of many of the measures of income mobility that have appeared in the literature has been on mobility between two periods, with notable exceptions including the works of Shorrocks (1978), Maasoumi and Zandvakili (1986), and Tsui (2009). Tsui (2009) offers a coherent framework to analyse multi-period income mobility. His approach is closely related to his previous work on multi-dimensional income inequality (Tsui, 1995, 1999), and provides also a decomposition of income mobility into structural and exchange components.

This paper proposes a different measurement framework for multi-period mobility, based on the concept of association, or dependence, in contingency tables. Realizing that the universally accepted notion of multi-period *immobility* is perfectly akin to the situation of contingency-table independence (i.e. complete lack of association between rows, e.g. people, and columns, e.g. time periods), our approach suggests measuring mobility by looking at the differences between observed income shares and their expected values under a situation of table independence.

Since these gaps may be positive or negative and we will assess mobility as inequality across these gaps, we will adopt an absolute inequality measurement framework and use absolute Lorenz curves (Moyes, 1987) for pre-orders. Previous pre-orders for contingency tables in the literature include the proposals by Joe (1985) and Greselin and Zenga (2004). However neither of them is intended or suited for measuring mobility as departure from contingency-table independence. We should also stress that, unlike Tsui (2009), we do not provide an axiomatic characterization of the mobility indices we introduce, since these are mainly adaptations of previously characterized absolute inequality indices.

To test the usefulness of the new multi-period income mobility indices that we propose, we look at income mobility in Europe, with the rotating panels of the EU-SILC dataset, covering the period 2005-2012. Besides computing mobility indices for the countries involved, we are interested in two specific questions: (1) whether "old" EU members exhibit more or less mobility than "new" EU members; and (2) whether the financial crisis of 2008 had an impact on income mobility in EU countries.

The paper is organized as follows. Section 2 presents the basic setting where multi-period income mobility is related to the association concept of contingency tables. The section discusses the extreme situation of complete immobility as independence between rows and columns in a contingency table, and then proceeds to lay out the desirable properties that mobility indices ought to satisfy in our framework. Section 3 introduces the multi-period mobility pre-order based on the concept of absolute Lorenz curves, which emerges naturally from the mobility concepts discussed in the previous section. Section 4 compares our approach with those of previous studies that proposed indices of multi-period mobility. Section 5 provides an empirical illustration based on EU-SILC data. Some concluding comments are given in Section 6.

2. Measuring multi-period mobility with contingency tables

2.1. Basic setting and the notion of immobility as contingency-table independence

Let $y_{it} \geq 0$ represent the income received by individual i at time t. Let Y represent the sum of incomes across people and across time:

$$\boldsymbol{Y \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it}}$$

where N is the total number of individuals in the panel and T the total number of periods. Define also a NxT matrix $\mathcal{S}$ whose typical element is s_{it}, defined as:

$$\boldsymbol{s_{it}} \equiv (\boldsymbol{y_{it}}/\boldsymbol{Y})$$

The margins of this matrix S are then

$$\boldsymbol{s_{i.}} \equiv \left(\sum_{t=1}^{T} \boldsymbol{y_{it}}/\boldsymbol{Y}\right)$$

and

$$\boldsymbol{s_{.t}} \equiv \left(\sum_{i=1}^{N} \boldsymbol{y_{it}}/\boldsymbol{Y}\right).$$

Finally let us also define a NxT matrix $\mathcal{W}$ whose typical element w_{ij} is expressed as $w_{ij} \equiv (s_{i.} \times s_{.t})$.

Note that y_{it} may be interpreted as an absolute frequency, in a contingency table with N rows and T columns. We call this $N \times T$ income table: $\mathcal{T}$. If so, then s_{it} is a relative frequency, $s_{i.}$ is a row marginal relative frequency and $s_{.t}$ is a column marginal relative frequency. Hence some elementary probability rules can be applied. For instance, if the income trajectories are independent of time periods then:

$$\boldsymbol{s_{it}} = \boldsymbol{s_{i.}}\boldsymbol{s_{.t}} = \boldsymbol{w_{ij}}$$

More precisely, we can establish the following proposition describing the shape of the individual distributions in the context of table independence:

Proposition 1: $\boldsymbol{s_{it}} = \boldsymbol{w_{it}}\ \forall \boldsymbol{i}, \boldsymbol{t}$ if and only if $\boldsymbol{y_{it}} = \boldsymbol{k_t}\boldsymbol{y_i}\ \forall \boldsymbol{i}, \boldsymbol{t}$, where $\boldsymbol{k_t} > \boldsymbol{0}$ and $\boldsymbol{y_i} > \boldsymbol{0}$.

Proof: See the Appendix.

According to Proposition 1, there is complete independence between people and time if and only if the income distribution in a given period can be expressed as a positive multiple of the income distribution in any other income distribution. Alternatively, independence is achieved if and only if, in the absence of any re-rankings, all distributions preserve the same level of relative inequality (as measured by any scale-invariant measure) across time.

Hence complete independence perfectly coincides with a lack of structural and exchange mobility, save for proportional transformations of the distributions. This is the same benchmark of immobility used previously in the literature (e.g. Shorrocks, 1978, and Maasoumi and Zandvakili, 1986).[2] Hence the degree of association or dependence between the rows and the columns, i.e. between the individuals and time, can serve as a metric for multi-period mobility in the population.

Several useful benchmarks of complete immobility can be derived from Proposition 1. The following one will be invoked in section 4, when we compare our proposal with previous approaches in the literature:

Corollary 1: Only in a situation of table independence it is the case that

$$\frac{\boldsymbol{s_{it}}}{\boldsymbol{s_{.t}}} = \frac{\boldsymbol{s_{it+\tau}}}{\boldsymbol{s_{.t+\tau}}} = \boldsymbol{s_{i.}}\ \forall(\boldsymbol{i}, \boldsymbol{t}, \boldsymbol{\tau}).$$

Corollary **1** states that only under table independence all the individual contributions to period income are equal across periods, and in turn these are all equal to the individual's lifetime income share (e.g. $s_{.i}$).

2.2. Mobility properties

Our proposal of desirable properties starts with some key definitions and then proceeds with properties that an index of multi-period mobility understood as departures from table independence should fulfil.

Let us also define a NxT matrix $\mathcal{V} \equiv NT(\mathcal{S} - \mathcal{W})$ whose typical element v_{ij} is a scaled absolute gap between observed shares and expected shares under independence, defined by $v_{ij} \equiv NT(s_{ij} - w_{ij})$. The reason why we multiply by NT will become apparent below.

[2] Tsui (2009), however, purges out any element of structural mobility from his mobility index, thereby using a different benchmark of complete immobility characterized by the lack of re-rankings, i.e. exchange mobility. Formally, his benchmark requires that: $s_{1t} \leq s_{2t} \leq \cdots \leq s_{Nt}\ \forall t \in [1, T]$. When we declare complete immobility, Tsui's benchmark also holds, however the reverse is not true.

We also define a mobility index mapping from the table of absolute gaps, $\mathcal{V}$, to the non-negative segment of the real line: $I(\mathcal{V})\colon \mathcal{V} \to \mathbb{R}_+$.

In what follows we rely on these absolute gaps, v_{it}, since we know by Proposition 1 that $v_{it} = 0\ \forall i,t$ if and only if there is table independence, i.e. complete immobility. Otherwise, some gaps will be positive while others will be negative. In this framework, *we will assess mobility as inequality across the gaps, since the gaps are only equal among each other (and equal to 0) whenever there is table independence.* Such an approach leads to the following definition of the property of complete immobility:

Complete immobility (IM): $I(\mathcal{V}) = 0$ if and only if $v_{it} = 0\ \forall i,t$.

Note that the mean value of the absolute gaps is zero. Therefore if we want to measure mobility as inequality across absolute gaps (since these can only be equal under complete immobility), we cannot rely on a relative approach. We have to adopt an absolute inequality measurement framework, which implies the use of absolute inequality indices and absolute Lorenz curves (Moyes, 1987) for pre-orders.

Notice also that, if we represent our data table via these gaps rather than through the use of the original shares s_{it}, we are gaining comparability in the sense that we are able to compare tables with different margins. However we have to take into account the fact that larger tables are bound to have smaller absolute gaps of the form $s_{it} - w_{it}$. But larger tables will also have more gaps to sum (in inequality functions that are additive with respect to functions of gaps). Nevertheless, depending on how the inequality index is defined, we might be violating some forms of the "population principle" (e.g. we may have indices "artificially" declaring larger tables to have lower inequality, hence less mobility). In order to solve this issue we suggest stating the following population principle in the case of tables:

Table population principle (TPP): If table $\mathcal{V}_2$ is obtained from table $\mathcal{V}_1$ by replicating its NT shares so that people are replicated $\lambda_N > 0$ times and periods are replicated $\lambda_T > 0$ then: $I(\mathcal{V}_1) = I(\mathcal{V}_2)$.

An interesting consequence of the dilution of gaps when tables grow in size, is that, in order to render gaps from tables with different sizes comparable, we need to ”blow up” all gaps by the table size, i.e. NT. Hence we need to measure mobility via the variables $v_{it} \equiv NT(s_{it} - w_{it})$. This is a necessary but insufficient requirement for making sure that mobility indices satisfy the TPP property.
We also want the mobility indices to satisfy a symmetry property:

Symmetry (S): If table $\mathcal{V}_B$ is obtained from table $\mathcal{V}_A$ by permutations of people (i.e. rows) or of time periods (i.e. columns), then: $I(\mathcal{V}_B) = I(\mathcal{V}_A)$.

Another desirable property of a multi-period mobility index is that it should react to regressive transfers. We suggest the following version of a regressive transfers property:

Sensitivity to regressive transfers among gaps (R): $I(\mathcal{V}_1) > I(\mathcal{V}_2)$ if $\mathcal{V}_1$ is obtained from $\mathcal{V}_2$ through a regressive transfer of $\delta > 0$ involving v_{it} and $v_{j\tau}$, with $v_{it} \leq v_{j\tau}$, so that $v_{it} - \delta < v_{j\tau} + \delta$.

Another desirable property to be mentioned is that of consistency. Such a property has been mentioned in the literature on inequality measurement in the case of bounded variables (see, Erreygers, 2009; Lambert and Zheng, 2010; Lasso de la Vega and Aristondo, 2012; Chakravarty et al., 2013; Silber, 2014) but it emerges also in our context. Basically, we are using v_{it}, but we could as well use $-v_{it}$. The choice between the two is essentially arbitrary. Therefore we should “impose” a property of consistency to the multi-period mobility indices:

Consistency (C): A table-mobility index is consistent if: $I(v_{11}^A, \dots, v_{NT}^A) > I(v_{11}^B, \dots, v_{NT}^B) \leftrightarrow I(-v_{11}^A, \dots, -v_{NT}^A) > I(-v_{11}^B, \dots, -v_{NT}^B)$.

2.3. Some indices measuring multi-period mobility

Are there indices that fulfil the properties previously mentioned? It turns out that all the classes of *consistent absolute inequality indices* proposed by Lambert and Zheng (2011), which include examples from Chakravarty et al. (2013), may be used to measure multi-period mobility. These indices include both rank-independent and rank-dependent families. Note that in defining these indices we omit the mean μ because $\mu \equiv \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} v_{it} = 0$. Here are some of the suitable indices:

- The variance:

$$\boldsymbol{\sigma = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(v_{it})^2} \tag{1}$$

- A family of generalized means:

$$\boldsymbol{M_\rho = \left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}|v_{it}|^\rho\right]^{\frac{1}{\rho}} \ \forall\rho > 1} \tag{2}$$

- An absolute Gini-related mobility index:

$$\boldsymbol{G_\rho = \left[\frac{1}{2(NT)^2}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{j=1}^{N}\sum_{\tau=1}^{T}\left|v_{it} - v_{j\tau}\right|^\rho\right]^{\frac{1}{\rho}} \ \forall\rho \geq 1} \tag{3}$$

Since several mobility indices are admissible, it is worth considering pre-orderings. Given that these mobility indices are all absolute inequality indices, we will base our analysis on the concept of absolute Lorenz curves.

3. Pre-orderings with an absolute Lorenz curve:

Let *A* and *B* be two populations. Following Moyes (1987) we define an *absolute Lorenz curve (ALC)*, $L: [0,1] \to (-1,0]$, which maps from population percentiles of v_{it} *in ascending order* to the actual cumulative values of $\frac{1}{NT}v_{it}$. Hence the ALC is:

$$\boldsymbol{L(p) \equiv \int_0^p v(q)dq} \tag{4}$$

where $v(q)$ is the quantile corresponding to percentile q. Note that in (4) μ is absent because $\mu = 0$.

We can now state a Lorenz-consistency condition akin to those used in the inequality literature (e.g. see Chakravarty, 2009):

Theorem 1: Table $\boldsymbol{A}$ exhibits more mobility than table $\boldsymbol{B}$ according to all mobility indices satisfying IM, S, TPP, R, and C, i.e. $\boldsymbol{I}[\boldsymbol{A}] > \boldsymbol{I}[\boldsymbol{B}]$, if and only if $\boldsymbol{L}^{\boldsymbol{A}}(\boldsymbol{p}) \leq \boldsymbol{L}^{\boldsymbol{B}}(\boldsymbol{p})\ \forall \boldsymbol{p} \in [\boldsymbol{0}, \boldsymbol{1}]$ and $\exists \boldsymbol{p} |\ \boldsymbol{L}^{\boldsymbol{A}}(\boldsymbol{p}) < \boldsymbol{L}^{\boldsymbol{B}}(\boldsymbol{p})$.

Proof: See the Appendix.

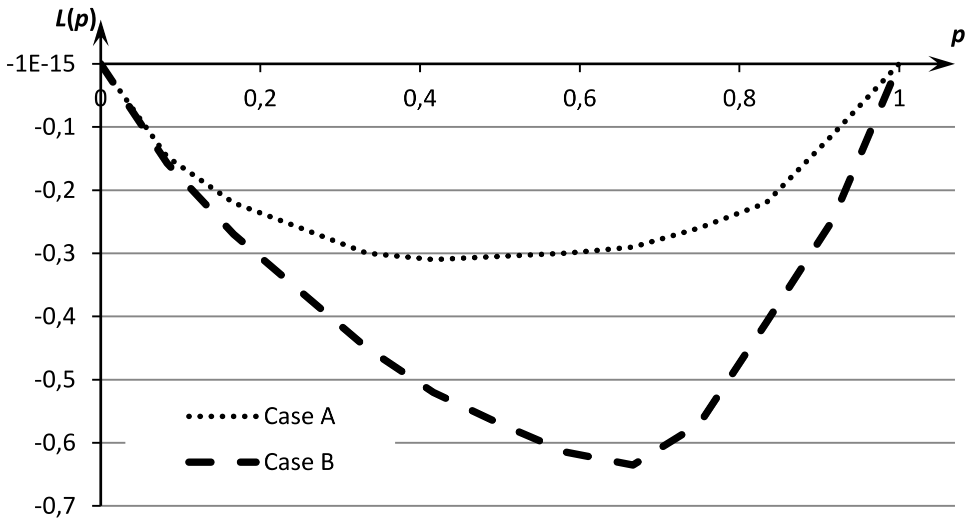
As an illustration consider the following two mobility tables, A and B with identical margins in Table 1.

Table 1: A simple illustration

	Period 1A	Period 2A	Period 3A	Period 1B	Period 2B	Period 3B
Person 1	0.01	0.04	0.2	0	0	0.25
Person 2	0.01	0.05	0.04	0	0	0.1
Person 3	0.05	0.05	0.1	0.2	0	0
Person 4	0.13	0.31	0.01	0	0.45	0

Their absolute Lorenz curve is drawn in **Erreur ! Source du renvoi introuvable.** below.

Figure 1: Two Absolute Lorenz curves



Hence any mobility index satisfying the properties stipulated in Theorem Theorem **1** should rank B as more mobile than A. This pre-ordering should allow us to compare not only tables with different sizes, but also tables with different margins, since we are mapping from absolute gaps. In fact, all gap tables of the form $\mathcal{V}$ have every margin equal to 0. In a sense, our definition of mobility is related to deviations from situations in which a table of gaps is full of zeroes.

4. Connection to previous measurement proposals in the literature

4.1. The Shorrocks multi-period mobility indices

Shorrocks (1978) defined a mobility index M based on a Lorenz-consistent inequality index I:

$$\boldsymbol{M_{SHORROCKS} = 1 - \frac{I(Y_1,\dots,Y_i,\dots Y_N)}{\sum_{t=1}^{T} s_{.t} I(y_{1t},\dots,y_{it},\dots,y_{Nt})}} \tag{5}$$

where $Y_i \equiv \sum_{t=1}^{T} y_{it}$. If we restrict the class I to that of scale-invariant indices then we can write (5) as:

$$M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t} I(s_{1t},\dots,s_{it},\dots,s_{Nt}) - I(s_{1.},s_{2.},\dots s_{N.})}{\sum_{t=1}^{T} s_{.t} I(s_{1t},\dots,s_{it},\dots,s_{Nt})}$$

$$\boldsymbol{M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t}[I(s_{1t},\dots,s_{it},\dots,s_{Nt}) - I(s_{1.},s_{2.},\dots s_{N.})]}{\sum_{t=1}^{T} s_{.t} I(s_{1t},\dots,s_{it},\dots,s_{Nt})}} \tag{6}$$

Invoking the scale invariance property again we can further rewrite:

$$M_{SHORROCKS} = \frac{\sum_{t=1}^{T} s_{.t}\left[I\left(\frac{s_{1t,}}{s_{.t}}\dots,\frac{s_{it,}}{s_{.t}},\dots,\frac{s_{Nt,}}{s_{.t}}\right) - I(s_{1.},s_{2.},\dots s_{N.})\right]}{\sum_{t=1}^{T} s_{.t} I(s_{1t},\dots,s_{it},\dots,s_{Nt})} \tag{7}$$

Finally we recall Corollary 1 and conclude that $M_{SHORROCKS} = 0$ if and only if there is table independence. That is, $M_{SHORROCKS}$ also considers table independence as the benchmark of complete immobility.

Regarding the differences between the approach proposed by Shorrocks (1978) and ours, we highlight that $M_{SHORROCKS}$ does not distinguish between tables characterized by a uniform distribution of lifetime shares, i.e. $s_{1.} = s_{2.} = \cdots = s_{N.}$. This is sensible in Shorrocks' framework given its interest in measuring mobility as equalization of lifetime incomes. Yet we can easily produce examples of pairs of tables sharing the same uniform column margin (lifetime shares) but differing in the level of inequality within their respective distributions of absolute gaps. Therefore our approach will distinguish within the set of matrices characterized by equalized lifetime shares those whose gaps indicate further departure from table independence. As an example, Table 2 provides two sets of distributions, A and B, both characterized by $s_{i.} = 0.25\ \forall i = 1, \dots, 4$. Clearly $M_{SHORROCKS}(A) = M_{SHORROCKS}(B)$. By contrast, if we compute the absolute Lorenz curves for both sets of distributions we will find that: $L^A(p) < L^B(p)\ \forall p$. Therefore any of our mobility indices declares A to be more mobile than B.

Table 2: Two mobility tables

	Period 1A	Period 2A	Period 3A	Period 1B	Period 2B	Period 3B
Person 1	0.25	0.0	0.0	0.1	0.1	0.05
Person 2	0.0	0.25	0.0	0.15	0.0	0.1
Person 3	0.0	0.0	0.25	0.05	0.15	0.05
Person 4	0.25	0.0	0.0	0.2	0.0	0.05

4.2. The Maasoumi and Zandvakili mobility indices

These indices start from Shorrocks' idea of comparing inequality of lifetime incomes against a weighted sum of snapshot income inequality across several periods, but they differ in: (1) Explicitly using the Generalized entropy family of inequality indices for I; (2) Using a generalized mean as a measure of lifetime income, i.e. $Z_i = \left[\sum_{t=1}^{T} a_t {y_{it}}^{\gamma}\right]^{\frac{1}{\gamma}}$, with $\sum_{t=1}^{T} a_t = 1$; With our notation and a few rearrangements we can express the indices as:

$$M_{MZ} = 1 - \frac{\sum_{i=1}^{N}\left[\left(\frac{Z_i}{\bar{Z}}\right)^{\delta} - 1\right]}{\sum_{t=1}^{T} s_{.t} \sum_{i=1}^{N}\left[\left(\frac{N s_{it}}{s_{.t}}\right)^{\delta} - 1\right]}, \delta \neq 0,1 \qquad (8)$$

where $\bar{Z} = \frac{1}{N}\sum_{i=1}^{N} Z_i$. Thanks to scale invariance we can actually use

$Z_i = \left[\sum_{t=1}^{T} a_t {s_{it}}^{\gamma}\right]^{\frac{1}{\gamma}}$. It is then easy to show that $\frac{Z_i}{\bar{Z}} = Ns_{i.}$ if and only if there is table independence. Since $\frac{Ns_{it}}{s_{.t}} = Ns_{i.}$ under those same circumstances, then it follows naturally that $M_{MZ} = 0$ if and only if there is table independence. Hence M_{MZ} is also measuring mobility with complete immobility as the same benchmark. We can establish similar results and conclusions for the two Theil versions of M_{MZ}.

However, again, we can find pairs of distributions for which M_{MZ} would yield the same value, whereas our approach clearly ranks one distribution as featuring more mobility as departure from table independence than the other distribution. For example, consider the choice $a_1 = a_2 = \ldots = a_T$. Now consider distributions A and B in Table 3. In distribution B every row-individual has different positive entries, but every row is a time-column permutation of any other row-individual. Meanwhile in distribution A every individual enjoys positive income in only one period. Moreover all individuals enjoy that same income (albeit in different periods in order to render all time margins positive). Then, clearly $Z_1 = Z_2 = \cdots = Z_N$ and $\sum_{t=1}^{T} s_{.t} \sum_{i=1}^{N} \left[\left(\frac{Ns_{it}}{s_{.t}}\right)^{\delta} - 1\right] > 0$ in both A and B. Therefore: $M_{MZ}(A) = M_{MZ}(B)$. By contrast, *all* our mobility indices would agree in deeming A more mobile than B, because one can easily show that $L^A(p) < L^B(p)\ \forall p$.

Table 3: Two other mobility tables

	Period 1A	Period 2A	Period 3A	Period 1B	Period 2B	Period 3B
Person 1	0.25	0.0	0.0	0.15	0.1	0.05
Person 2	0.0	0.25	0.0	0.15	0.05	0.1
Person 3	0.0	0.0	0.25	0.1	0.15	0.05
Person 4	0.25	0.0	0.0	0.05	0.1	0.15

4.3. The mobility indices of Tsui (2009)

Tsui (2009) derived a multi-period income mobility index which, in our notation is expressed as:

$$M_{TSUI} = \frac{\rho}{N} \sum_{i=1}^{N} \left[\prod_{t=1}^{T} \left(\frac{Ns_{it}}{s_{.t}}\right)^{c_t} - 1\right] \tag{9}$$

where ρ and c_t are parameters.[3]

We recall that Corollary 1 states that only under independence: $\frac{s_{it}}{s_{.t}} = s_{.i}\ \forall(i,j,t)$. Then, clearly, for $c_t \neq 0$, $M_{TSUI} = 0$ if and only if $s_{it} = \frac{1}{NT}\ \forall i,t,$ i.e. if all the shares are equal to each other. While this situation would certainly qualify as one of table independence, it is not the only such situation. Therefore other situations of table independence, e.g. any in which $s_{it} = s_{i.}s_{.t}$, will not minimize the value of M_{TSUI}. Hence this index does not set table independence generally as its benchmark of complete immobility. Implicitly, M_{TSUI} considers any common growth factor between two periods as a source of mobility. By contrast, in our proposed framework, if the only difference between all snapshot income distributions is a common growth factor, i.e. a multiplication in period 2 of each period 1 income by the same positive scalar, then we are in a situation of complete immobility and table independence.

Moreover, again, we can find pairs of distributions for which M_{TSUI} would yield the same value, whereas our approach clearly ranks one distribution as featuring more mobility as departure from table independence than the other distribution. For example, note that M_{TSUI} yields the same value for all tables characterized by rows in which every individual features at least one null income, i.e. $\forall N: \exists t | s_{it} = 0$.
Now consider distributions A and B in
Table **4**. In distribution B every individual has no income in one period. Meanwhile in distribution A every individual enjoys positive income in only one period. Therefore: $M_{TSUI}(A) = M_{TSUI}(B)$. By contrast, *all* our mobility indices would agree in deeming A more mobile than B, because one can easily show that $L^A(p) < L^B(p)\ \forall p$.

Table 4: A third illustration of mobility tables

	Period 1A	**Period 2A**	**Period 3A**	**Period 1B**	**Period 2B**	**Period 3B**
Person 1	0.25	0.0	0.0	0.0	0.1	0.1
Person 2	0.0	0.25	0.0	0.15	0.0	0.1
Person 3	0.0	0.0	0.25	0.1	0.15	0.0

[3] See Tsui (2009) for more details on the choice of these parameters.

Person 4	0.25	0.0	0.0	0.2	0.0	0.1

In summary, even though some of the proposals from the literature agree with ours on certain key axioms (mainly IM, which is satisfied by the proposals of both Shorrocks, and Maasoumi-Zandvakili), the three proposals are inconsistent with our measurement framework. This should not come as a major surprise, or be deemed an indictment on the previous literature, since none of the reviewed contributions had as its stated purpose the measurement of mobility as departure from table independence.

5. An empirical application: multi-period mobility in European countries

5.1. Data description

The empirical analysis has been performed using income data from the EU-SILC study, which was launched in 2003. In the first year, however, it covered only 6 countries. In subsequent years, the number of countries underwent a gradual increase. Thus, currently, it is carried out in all member states of the European Union and several European countries outside the EU, including Switzerland, Norway and Turkey. However, our analysis will concentrate only on European Union countries.

In most countries, households participating in the EU-SILC are surveyed on the basis of four-year rotational panels. This means that each year about one-fourth of the whole sample is replaced by a new group of households. The consequence of such method of constructing the sample is the availability of panel data for periods no longer than 4 years.

Due to the low number of countries participating in the EU-SILC survey at the beginning, income mobility analysis was performed for selected countries of the European Union for the period 2005-2012. This period includes two non-overlapping 4-year sub-periods: 2005-2008 and 2009-2012. However, since we have a rotating panel, it is possible to carry out the analysis for 4-year periods which partially overlap. This allows for a more detailed assessment of the impact of the data coming from consecutive rounds of EU-SILC study.

In the analysis, we used data on income of individuals in households (variable PY010G – gross employee cash or near cash income) available in the longitudinal personal data file. This income was recorded for all current household members aged 16 and above (for details see Description of target variables, 2008; and more recent documents).

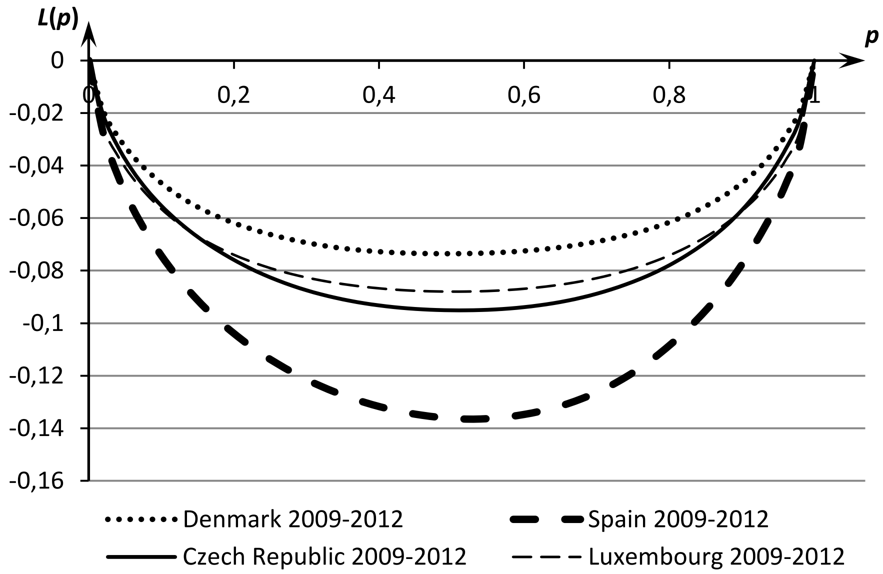
Income values are expressed in Euros, which means that for countries outside the euro zone income levels have been converted at current exchange rates.

5.2. Results

According to Theorem 1 we can rank countries with respect to the income mobility on the basis of the Lorenz-consistency condition. If $\boldsymbol{L^A(p) \leq L^B(p)}\ \forall \boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$ and $\exists \boldsymbol{p}|\ \boldsymbol{L^A(p) < L^B(p)}$ for countries A and B, respectively, then all proposed indexes will judge income in country A to be relatively more mobile than income in country B.

To illustrate this relationship, absolute Lorenz curves for selected countries are presented in **Erreur ! Source du renvoi introuvable.**.

Figure 2: Absolute Lorenz Curves for Denmark, Czech Republic, Luxembourg and Spain in 2009-2012



The curves in **Erreur ! Source du renvoi introuvable.** indicate that Denmark is the least mobile country so that its curve "dominates" those of the other three countries (Luxembourg, the Czech Republic and Spain). On the other hand Spain is the most mobile country and its curve is "dominated" by that of the Czech Republic, Luxemburg and Denmark. The curves of the Czech Republic and Luxembourg intersect so that the relative assessment of their income mobility depends on the choice of the mobility index.

In what follows we use only the G_ρ index with $\rho = 1$ for the assessment of income mobility. As noted earlier, in such a case, this index is in fact the Gini coefficient of inequality. This index can then be interpreted as the expected value of the absolute differences between all the elements of the matrix $\mathcal{V}$.

The results of the assessment of income mobility levels are shown in Table 5.

Table 5: Income mobility in selected European Union countries

Country	Income mobility in following periods					Country characteristics
	2005-2008	2006-2009	2007-2010	2008-2011	2009-2012	
Austria	0.196 (0.006)	0.222 (0.008)	0.228 (0.014)	0.208 (0.007)	0.171 (0.006)	Euro zone*
Belgium	0.139 (0.006)	0.187 (0.006)	0.183 (0.006)	0.182 (0.006)	0.151 (0.006)	Euro zone *
Bulgaria		0.289 (0.009)	0.249 (0.008)	0.233 (0.006)	0.205 (0.006)	New EU country**
Cyprus	0.124 (0.005)	0.119 (0.005)	0.123 (0.005)	0.127 (0.006)	0.116 (0.006)	New EU country**
Czech Republic	0.170 (0.004)	0.171 (0.004)	0.166 (0.004)	0.181 (0.005)	0.149 (0.004)	New EU country**
Denmark	0.103 (0.004)	0.123 (0.005)	0.114 (0.004)	0.115 (0.004)	0.118 (0.005)	
Estonia	0.210 (0.011)	0.243 (0.007)	0.238 (0.007)	0.216 (0.006)	0.206 (0.006)	New EU country**
France	0.139 (0.003)	0.184 (0.003)	0.180 (0.005)	0.169 (0.004)	0.135 (0.002)	Euro zone *
Greece		0.139 (0.008)	0.162 (0.006)	0.181 (0.007)	0.208 (0.008)	Euro zone *
Hungary	0.226 (0.007)	0.248 (0.007)	0.219 (0.006)	0.230 (0.006)	0.191 (0.005)	New EU country**
Italy		0.171 (0.004)	0.180 (0.004)	0.199 (0.004)	0.172 (0.019)	Euro zone *
Latvia		0.212 (0.008)	0.228 (0.007)	0.240 (0.007)	0.221 (0.007)	New EU country**
Lithuania	0.177 (0.007)	0.182 (0.006)	0.223 (0.007)	0.219 (0.008)	0.207 (0.006)	New EU country**
Luxembourg	0.141 (0.004)	0.169 (0.011)	0.173 (0.010)	0.160 (0.004)	0.143 (0.004)	Euro zone *
Malta		0.187 (0.008)	0.160 (0.007)	0.164 (0.007)	0.129 (0.006)	New EU country**
Poland	0.207 (0.004)	0.226 (0.004)	0.219 (0.006)	0.219 (0.005)	0.191 (0.004)	New EU country**
Portugal		0.198 (0.009)	0.198 (0.008)	0.210 (0.009)	0.163 (0.006)	Euro zone *
Romania			0.161 (0.005)	0.132 (0.004)	0.131 (0.005)	New EU country**
Slovakia	0.204 (0.007)	0.194 (0.005)	0.186 (0.005)	0.181 (0.006)	0.173 (0.006)	New EU country**
Slovenia	0.148 (0.004)	0.163 (0.004)	0.152 (0.004)	0.156 (0.004)	0.136 (0.003)	New EU country**
Spain	0.188 (0.004)	0.225 (0.005)	0.212 (0.004)	0.212 (0.004)	0.190 (0.004)	Euro zone *
Sweden	0.137 (0.004)	0.140 (0.005)	0.139 (0.004)	0.145 (0.005)		
United Kingdom	0.186 (0.011)	0.210 (0.006)	0.211 (0.007)	0.234 (0.010)	0.187 (0.008)	

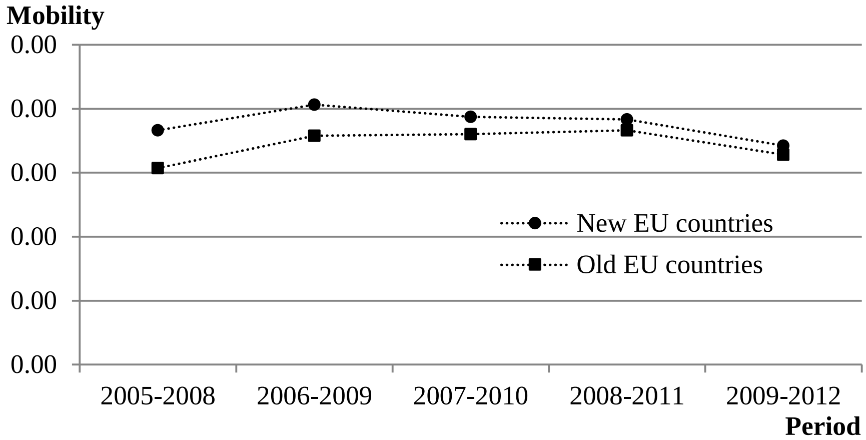
Estimated standard errors in parentheses (based on 1,000 bootstrap samples)

* Countries belonging to the euro zone before January 1, 2005.
** Countries that joined the European Union after January 1st 2004.

Source: own calculations

When looking at the relative levels of income mobility, it is worth paying attention to two observations. First it appears that on average the level of income mobility is higher among the new EU member states (states which joined the European Union after January 1, 2004). The average level of income mobility among the old and new EU members is illustrated in **Erreur ! Source du renvoi introuvable.**.

Figure 3: Comparison of average income mobility in "Old" and "New" European Union Countries



Although the differences between the old and the new EU gradually decreased over time, income mobility is systematically higher in the new EU countries. One may think of various reasons for such a higher mobility. Firstly, these new EU countries are characterized by a lower average level of income and, at the same time, a generally higher rate of economic growth. In conjunction with the continued process of economic transformation, such a combination may lead to major changes in relative incomes and a lower stability. Another factor which could play an important role in income mobility assessment is a floating exchange rate of national currencies. During the financial crisis, currencies of the new EU countries were significantly devaluated and this affected the relative incomes (in Euros). Poland is a good illustration. Despite a positive rate of GDP growth and increasing average wages (as expressed in national currency), the average income in Euro terms declined between 2009 and 2010. A similar situation (but involving declines in GDP per capita) occurred in other countries. Detailed information on average income levels in the different countries is presented in Table 6.

Table 6: Average personal income

Country	Average personal gross income in consecutive years [EUR]							
	2005	2006	2007	2008	2009	2010	2011	2012
Austria	21033	19761	20234	21228	21549	22709	23031	25593
Belgium	24201	23870	23653	24017	25809	26626	27565	28822
Bulgaria		1183	1353	2251	2819	2682	2861	2898
Cyprus	13547	12063	12683	13289	14621	16130	18069	20443
Czech Republic	4714	5146	5810	6561	7961	7479	7980	8710
Denmark	28971	27432	29091	30617	32459	33087	33859	35140
Estonia	4402	4292	4533	5935	6887	6211	6664	7338
France		17983	18193	19326	19840	20170	21058	21944
Greece			13470	14525	15435	14906	13930	12373
Hungary	3832	4166	4342	4811	5107	4658	4953	5105
Italy			16822	17036	17484	16984	17556	17551
Latvia			3538	5346	6522	5136	4995	5279
Lithuania	3017	3540	4507	5328	6009	4786	4295	5327
Luxembourg	35797	36130	35923	36543	29833	33199	37391	43045
Malta		7223	8603	9267	12121	12622	14112	14565
Poland	3485	4299	5014	6001	7216	6068	6791	6970
Portugal			10059	10757	11179	11380	11350	11059
Romania			2672	3399	3689	3302	3345	3471
Slovakia	3031	3341	3913	4775	5838	5974	6320	6630
Slovenia	10032	9671	10043	10868	12267	12698	13406	13829
Spain	14609	14460	14452	15432	15534	14799	14703	14358
Sweden	21509	21537	22392	23387	22904	20859	24213	
United Kingdom	27966	27859	29546	26332	22893	24260	23997	25107

Source: own calculations

Data on changes in average income levels will also help in discussing the second issue which concerns the impact of the financial crisis (which began in 2008) on the level of income mobility in the various countries. Among countries particularly affected by the crisis we can mention Greece, Spain and Portugal. We do not add to this group other countries – especially Latvia and Lithuania – in spite of the fact that the impact of the crisis on income levels was also very serious in their case. In these countries, however, an additional factor influencing the change in average income was the exchange rate. To neutralize the role played by this factor, the analysis concentrates on countries which belonged to the euro zone at the beginning of the period (January 1, 2005). The results are presented in **Erreur ! Source du renvoi introuvable.**.

Figure 4. Comparison of average income mobility in Greece, Portugal and Spain and other euro zone countries

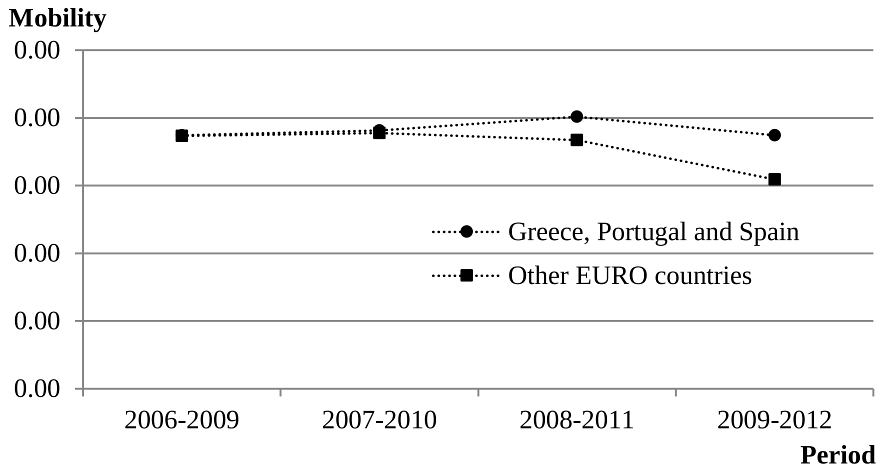


Figure 4 shows that while initially Greece, Portugal and Spain had levels of income mobility similar to that of other Euro countries, in subsequent years the trends were different. The relatively high average level of income mobility gradually decreased in the group of other countries, while remaining high in Greece, Portugal and Spain. In these countries, the crisis resulting from the significant level of public debt led to budgetary adjustments. The consequences of these adjustments were observed in the following years, in terms of both income levels and mobility. The higher levels of income mobility observed in Greece, Portugal and Spain suggest a lack of stability and income insecurity (like a higher risk of losing a job or bankruptcy).

6. Concluding comments

Although some suggestions have been made in the past to measure multi-period income mobility, most studies of income mobility, in particular those with an empirical analysis, considered only two periods. Initially, the basic idea of the approach proposed in the present paper was that, in the same way as the measurement of income inequality amounts to comparing population shares with income shares, indices of income mobility could be considered as comparing " a priori" with "a posteriori" income shares. A typical "a posteriori" share would refer to the income share of some individual at a given time in the total income of all individuals over the whole period analysed. The corresponding "a priori" share would be the hypothetical income share in the total income of society over the whole accounting period that an individual would have had at a given time, *had there been complete independence between the individuals and the time periods*.

Previous proposals of multi-period mobility in the literature also identified the benchmark of complete immobility with independence between individuals and time periods, often implicitly. However, as we showed in the paper, these approaches, unlike our proposal, measure, explicitly or implicitly, alternative notions of mobility, different from our concept of mobility as departure from contingency-table independence.

A thorough examination of such an approach based on shares' comparisons showed, however, that one should be more careful, and that a more appropriate way of consistently measuring multi-period mobility should focus on the absolute rather than the traditional (relative) Lorenz curve and that the relevant variable to be accumulated should be the difference between the "a priori" and "a posteriori" shares previously defined. Moving from an ordinal to a cardinal approach to measuring multi-period mobility, we then proposed classes of mobility indices based on absolute inequality indices. For the sake of simplicity we only used one index in the empirical illustration of our paper, the one which is directly related to the absolute Gini index.

The empirical analysis seems to have vindicated our approach because it clearly showed that income mobility was higher in the new EU countries (those that joined the EU in 2004 and later). We also observed that income mobility after 2008 was higher in three countries that were particularly affected by the financial crisis: Greece, Portugal and Spain. Additional work is probably needed to further justify the use of the new approach to multi-period income mobility that has been proposed in this paper.

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Appendix: Proofs

Proof of Proposition 1:

Sufficiency: if $y_{it} = k_t y_i$ then: $s_{it} = \frac{k_t y_i}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]}$, $s_{i.} = \frac{y_i \sum_{t=1}^{T} k_t}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]} = \frac{y_i}{\sum_{i=1}^{N} y_i}$ and $s_{.t} = \frac{k_t \sum_{i=1}^{N} y_i}{[\sum_{t=1}^{T} k_t][\sum_{i=1}^{N} y_i]} = \frac{k_t}{\sum_{t=1}^{T} k_t}$. Then clearly: $s_{it} = s_{i.} s_{.t}$.

Necessity: if $s_{it} = s_{i.} s_{.t}$, then: $\frac{y_{it}}{Y} = \frac{\sum_{t=1}^{T} y_{it}}{Y} \frac{\sum_{i=1}^{N} y_{it}}{Y}$, which leads to: $y_{it} = \frac{\sum_{t=1}^{T} y_{it} \sum_{i=1}^{N} y_{it}}{Y}$. Setting $y_i = \sum_{t=1}^{T} y_{it}$and $k_t = \frac{\sum_{i=1}^{N} y_{it}}{Y}$, it is clear to see that independence requires y_{it} to be of the form $k_t y_i$.

Proof of Theorem 1:

Satisfaction of IM and absolute Lorenz dominance:

Let A be a table of gaps characterized by $\exists (i,t) | v_{it} \neq 0$ (which requires, in fact, that at least one gap is negative and one gap is positive), and B be a table of gaps characterised by $v_{it} = 0 \; \forall i, t$. Then if I fulfils IM it must be the case that: $I[A] > I[B] = 0$. Meanwhile $L^B(p) = 0 \; \forall p \in [0,1]$, whereas $\exists p | L^A(p) < 0$. Therefore any index satisfying IM ranks a table characterized by complete immobility as less mobile than any other table if and only if the absolute Lorenz curve of the completely immobile table is nowhere below that of the other table, which is bound to be the case since the absolute Lorenz curve of a completely immobile table is a straight line overlapping with the horizontal axis.

Satisfaction of S and absolute Lorenz dominance:

This is straightforward since the absolute Lorenz curve requires arranging all gaps for accumulation in ascending order of value. A permutation of individual-rows and/or time-columns would not alter the final arrangement in ascending order. Therefore if A is obtained from B through a sequence of permutations of rows and columns, any index satisfying S would yield $I[A] = I[B]$ by definition, while at the same time we would get: $L^A(p) = L^B(p) \; \forall p \in [0,1]$.

Satisfaction of TPP and absolute Lorenz dominance:

Let A be obtained from B through a replication of gaps such that the N rows of B are multiplied λ_N times and the T columns of B are multiplied λ_T times. By definition, any index satisfying TPP would yield $I[A] = I[B]$. Meanwhile, note that: $v_{it}^A = \lambda_N N \lambda_T T \left(\frac{s_{it}^B - w_{it}^B}{\lambda_N \lambda_T}\right) = NT(s_{it}^B - w_{it}^B) = v_{it}^B$. Therefore we would get: $L^A(p) = L^B(p)\ \forall p \in [0,1]$.

Satisfaction of C and absolute Lorenz dominance:

Let table –A be obtained from A by multiplying each of its elements by -1. Same for tables B and –B. We need to prove that if it is true that $I[A] > I[B]$, if and only if $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$, then it should also be the case that $I[-A] > I[-B]$, if and only if $L^{-A}(p) \leq L^{-B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$, and if the index is consistent.

By definition, we know that if I is consistent, then $I[A] > I[B]$ if and only if $I[-A] > I[-B]$. Hence what we really need to prove is whether the absolute Lorenz curve is consistent, i.e. $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$ if and only if $L^{-A}(p) \leq L^{-B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$.

Note that both $L^A(p)$ and $L^{-A}(p)$ rely on the same gaps. The difference being that the ordered sequence of gaps in A is the exact opposite of the ordered sequence of gaps in –A. Hence $L^A(p) = L^{-A}(1-p)$ and $L^B(p) = L^{-B}(1-p)$. Then it clearly follows that: $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$ if and only if $L^{-A}(p) \leq L^{-B}(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$.

Satisfaction of R and absolute Lorenz dominance:

We know from Moyes (1987, proposition 3.1, p. 205) that $I[A] > I[B]$ for any inequality index satisfying R, if and only if $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$.

An alternative proof requires proving: (1) that if A is obtained from B through a sequence of regressive transfers (which means $I[A] > I[B]$ for any inequality index satisfying R) then it will also be the case that $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$; (2) and that if we have $L^A(p) \leq L^B(p)\ \forall p \in [0,1]$ and $\exists p|\ L^A(p) < L^B(p)$ then we can obtain $L^A(p)$ from $L^B(p)$ through a sequence of regressive transfers (which would then mean $I[A] > I[B]$ for any inequality index satisfying R).

Part (1) is easy to prove by realising that any intermediate regressive transfer involving percentiles q and r, such that $q < r$, will lead to a new absolute Lorenz curve lying below the previous one between percentiles q and r, while overlapping elsewhere. Part (2) requires a sequence like the following: start with the percentile ϵ (where ϵ is very close to 0). We define the quantity $q(\epsilon) \equiv -[L^A(\epsilon) - L^B(\epsilon)]$ which is the amount that we would need to subtract from the lowest gap in B in order to reach the Lorenz vertical coordinate of A at ϵ. We can then implement a regressive transfer of $q(\epsilon)$ out of $v^B(\epsilon)$ and into any of the gaps belonging in the closest percentile to the right of ϵ, i.e. $\epsilon + \theta$. Naturally $L^B(\epsilon + \theta)$ will not be affected by this transfer. But then the next step is to transform $L^B(\epsilon + \theta)$ into $L^A(\epsilon + \theta)$. Again, we define: $q(\epsilon + \theta) \equiv -[L^A(\epsilon + \theta) - L^B(\epsilon + \theta)]$. Then we can subtract $q(\epsilon + \theta)$ from either one or a combination of gaps within $\epsilon + \theta$ and dump it into one or a combination of gaps within the closest percentile to the right of $\epsilon + \theta$ (i.e. this last regressive transfer may actually be a subsequence of regressive transfers). The same procedure can be repeated until reaching the last percentile.

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